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Code Number 041/1/2



**INDIAN SCHOOL MUSCAT  
THIRD PRELIMINARY EXAMINATION  
MATHEMATICS**

CLASS: XII  
01.02.2018

Sub. Code: 041  
SET-2

Time Allotted: 3 Hrs  
Max. Marks: 100

**General Instructions:**

- (i) All questions are compulsory.
- (ii) This question paper contains 29 questions.
- (iii) Question 1- 4 in Section A are very short-answer type questions carrying 1 mark each.
- (iv) Question 5-12 in Section B are short-answer type questions carrying 2 marks each.
- (v) Question 13-23 in Section C are long-answer-I type questions carrying 4 marks each.
- (vi) Question 24-29 in Section D are long-answer-II type questions carrying 6 marks each.

**SECTION – A ( 4 x 1 = 4)**  
Questions 1 to 4 carry 1 mark each

1. Find the vector equation of a plane at which is at a distance of 5 units from the origin and the normal to the vector  $(3\hat{i} + 5\hat{j} - 6\hat{k})$ . 1
2. Find the points where the function  $f(x) = [x]$ ,  $-1 \leq x < 3$  is not differentiable. 1
3. Find  $\frac{dy}{dx}$ , if  $y = \sin^{-1}(\cos x)$  1
4. Find  $x$ , if  $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$  is singular. 1

**SECTION – B ( 8 x 2 = 16)**  
Questions 5 to 12 carry 2 marks each

5. Find  $\lambda$  if the vectors  $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$  and  $\vec{b} = 3\hat{i} - 2\hat{j} + \lambda\hat{k}$  are perpendicular 2
6. A balloon which is always remains spherical, has a variable radius. Find the rate at which its volume is increasing w.r.t its radius when the radius is 7cm. 2
7. Evaluate:  $\int \tan^{-1}x dx$  2
8. Find the points on the curve  $y = x^3 - 11x + 5$  at which the tangent has the equation  $y = x - 11$  2
9. If  $y = \begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$ , find  $\frac{dy}{dx}$ . 2

10. Find the angle between the pair of the lines given by 2  

$$\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3} \text{ and } \frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$$
11. There are two types of fertilizers  $F_1$  and  $F_2$ .  $F_1$  consists of 10% of nitrogen and 6% phosphoric acid and  $F_2$  consists of 5% of nitrogen and 10% phosphoric acid. After testing the soil conditions a farmer finds that she needs at least 14 kg of nitrogen and 14kg of phosphoric acid for her crops, If  $F_1$  costs Rs6/kg and  $F_2$  costs Rs5/Kg. Formulate the problem so that nutrient requirements are met at a minimum cost. 2
12. Ten eggs are drawn successively with replacement from a lot containing 20% defective eggs. Find the probability that there is exactly one defective egg. 2

SECTION – C (11 x 4 = 44)  
 Questions 13 to 23 carry 4 marks each

13. If  $y = \sqrt{(x-2)(x+3)(x+5)}$ , find  $\frac{dy}{dx}$ . 4

OR

Find the value of k, if  $f(x) = \begin{cases} \frac{1-\cos kx}{x \sin x}, & \text{if } x \neq 0 \\ \frac{1}{2}, & \text{if } x = 0 \end{cases}$  is continuous at  $x = 0$ .

14. Evaluate the integral:  $\int_1^4 \{|x-1| + |x-2| + |x-4|\} dx$ . 4  
 (OR)

Evaluate the integral:  $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ .

15. Integrate:  $\int \frac{3x-2}{(x+1)^2(x+3)} dx$ . 4

16. Solve The Following:  $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$  4

17. Prove that  $\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3$  4

OR

Prove that  $\begin{vmatrix} a+bx & c+dx & p+qx \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix} = (1-x^2) \begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$

18. Solve :-  $(1+x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$  given  $y = 0$  when  $x = 1$  4

19. If with reference to the right handed system of mutually perpendicular unit vectors  $\hat{i}, \hat{j}$  and  $\hat{k}$ ,  $\vec{a} = 3\hat{i} - \hat{j}$ ,  $\vec{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}$ , then express  $\vec{\beta}$  in the form  $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$  where  $\vec{\beta}_1$  is parallel to  $\vec{a}$  and  $\vec{\beta}_2$  is perpendicular to  $\vec{a}$ . 4

20.  $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$ ,  $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ ,  $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$ , Find a vector  $\vec{d}$  which is perpendicular to both  $\vec{a}$  and  $\vec{b}$  &  $\vec{c} \cdot \vec{d} = 15$  4

21. In an examination, an examinee either guesses or copies or knows the answer of multiple choice question with 5 choice. The probability that he makes a guess is  $1/3$  and the probability he copies is  $1/6$ . Find the probability that his answer is correct, given that he copied it is  $1/8$ . Find the probability that he knew the answer to the question, given that he correctly answered it. 4

22. Two cards are drawn successively, with replacement from a well shuffled deck of 52 cards. Find the probability distribution of the number of spades. 4

23. Maximise  $Z = 3x + 5y$ , subject to the constraints: 4  
 $x + 2y \leq 20$ ,  $x + y \leq 15$ ,  $y \leq 5$ ,  $x, y \geq 0$ .

SECTION – D ( 6 x 6 = 36)  
 Questions 24 to 29 carry 6 marks each

24. Show that the semi vertical angle of right circular cone of given surface area and maximum volume is  $\sin^{-1} \frac{1}{3}$ . 6

OR

The length of the sides of an isosceles triangle are  $9+x^2, 9+x^2$  and  $18-2x^2$  units. Calculate the value of  $x$  which makes the area maximum. Also find the maximum area of the triangle.

25. Let  $f: \mathbf{R}_+ \rightarrow [-5, \infty]$  be a function defined as  $f(x) = 9x^2 + 6x - 5$ . show that 6  
 $f$  is invertible. Find the inverse of  $f$ .

OR

Consider the operation  $*$  on  $\mathbf{Q} - \{-1\}$  defined by  $a*b = a + b + ab$ .  $a, b \in \mathbf{Q} - \{-1\}$ . Prove that  $*$  is binary. 2) Is  $*$  commutative? 3) Is  $*$  associative 4) does the identity for  $*$  exist?

If yes, find the identity. 5) Are elements of  $\mathbf{Q}$  invertible? if so find the inverse of a rational numbers other than  $-1$ .

26. Find the area of the region enclosed between the two circles  $x^2 + y^2 = 4$  and  $(x - 2)^2 + y^2 = 4$  6

OR

Using integration, find the area of the following region:  $\{(x, y) : |x - 1| \leq y \leq \sqrt{5 - x^2}\}$

27. Solve:-  $\left\{ x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right) \right\} y dx = \left\{ y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right) \right\} x dy$  6

28. Find the equation of the plane through the intersection of the planes  $x + y + z = 1$  and  $2x + 3y + 4z = 5$  which is perpendicular to the plane  $x - y + z = 0$ . 6
- OR
- Find the length and the foot of the perpendicular from the point  $(1, 3, 4)$  to the plane  $2x - y + z + 3 = 0$ , also find image point.
29. The sum of three numbers is 6. If we multiply the third number by 2 and add the first number to the result, we get 7. By adding second and third numbers to three times the first number, we get 12. Using matrices, find numbers. 6

**End of the Question Paper**